SUPPLEMENTARY INFORMATION

Imperfect drug penetration leads to spatial monotherapy and rapid evolution of multi-drug resistance

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Part I
Basic results of the viral dynamics model

1 Deterministic model

In the absence of mutation or migration, the dynamics for a virus of strain i, present in compartment j, can be described using the basic viral dynamics equations (1):

$$\begin{aligned} \dot{x_j} &= \lambda_j - \beta_{ij} x_j v_{ij} - d_x x_j \\ \dot{y_{ij}} &= \beta_{ij} x_j v_{ij} - d_y y_{ij} \\ \dot{v_{ij}} &= k y_{ij} - d_v v_{ij} \end{aligned} \tag{1}$$

where x_j, y_{ij} and v_{ij} are the populations of uninfected cells, cells infected with strain i and free virus of strain i, respectively - all in compartment j. Uninfected host cells die at rate d_x and are produced at rate λ_j. These cells become infected by strain i at rate β_{ij}. Infected cells die at rate d_y and produce free virus at rate k. Free virus is cleared at rate d_v. Implicit in these parameter choices is the assumption that compartments differ only in the rate at which uninfected cells are produced, and viral strains differ only in the rate at which they infect new cells.

The *basic reproductive ratio* (i.e. the number of new infections generated by an infected cell before it dies in a totally susceptible population of host cells) for this model is $R_0^{ij} = \lambda_j \beta_{ij} k/(d_x d_y d_v)$ (Ref (1)).

This system can be simplified by assuming that the population of free virus instantaneously reaches an equilibrium with respect to the population of infected cells. This separation of timescales is valid when we are not interested in short term fluctuations, because the dynamics of the virus tend to be much faster than those of cells (1; 2). We therefore set $\dot{v_{ij}} = 0$ and get $v = (k/d_v)y$, and by defining $B_{ij} = \beta_{ij} k/d_v$ we reduce the model to two equations tracking only cells:

$$\begin{aligned} \dot{x_j} &= \lambda_j - B_{ij} x_j y_{kj} - d_x x_j \\ \dot{y_{ij}} &= B_{ij} x_j y_{ij} - d_y y_{ij} \end{aligned} \tag{2}$$

There are two steady state solutions to this system of equations when only a single strain is present: When $R_0^{ij} \equiv \lambda_j B_{ij}/(d_x d_y) < 1$, there is no infection, and when $R_0^{ij} > 1$, the infection reaches a steady state level:

$$\{x_j^*, y_{ij}^*\} = \begin{cases} \left\{ \dfrac{\lambda_j}{d_x}, 0 \right\} \equiv \{N_j, 0\} & \text{if } R_0^{ij} < 1 \\ \left\{ \dfrac{N_j}{R_0^{ij}}, \dfrac{N_j d_x}{d_y} \left(1 - \dfrac{1}{R_0^{ij}} \right) \right\} \equiv \left\{ \dfrac{N_j}{R_0^{ij}}, K_j^i \right\} & \text{if } R_0^{ij} > 1 \end{cases} \tag{3}$$

When there is more than one virus strain in a single compartment at a given time, the equations can easily be modified and new steady state solutions and stability conditions derived. The result is that only one virus strain ever remains in the compartment at steady state (competitive exclusion) (1). This is the strain with the highest R_0 value.

These steady state equations give rise to terms we will frequently use throughout the paper. The total number of uninfected host cells that a compartment j contains when there is no virus present is called the *compartment size* and is given by N_j. The equilibrium number of infected cells of type i that are present in a compartment when $R_0^{ij} > 1$ for strain i and $R_0^{ij} > R_0^{kj}$ for all $k \neq i$ is termed the *carrying capacity* and is denoted by K_j^i.

This system can be extended to account for mutation and migration, along with the presence of multiple strains:

$$\begin{aligned} \dot{x_j} &= \lambda_j - x_j \sum_k B_{kj} y_{kj} - d_x x_j \\ \dot{y_{ij}} &= x_j \sum_k \mu_{ki} B_{kj} y_{kj} - (d_y + \sum_q m_{jq}) y_{ij} + \sum_q m_{qj} y_{iq} \end{aligned} \tag{4}$$

where μ_{ki} is the probability per infection event that strain k mutates to strain i, and m_{qj} is the rate of migration from compartment q to compartment j. Note that we have ignored the migration of uninfected cells, since it is not important for the evolutionary process we are interested in. Because we are only tracking cells, and not virus, we have implicitly assumed that it is infected cells that migrate. This assumption should have only miminal influence on our results, because while virus numbers are much larger than those of infected cells, the establishment probability starting from a single virion is much lower.

This system no longer yields a tractable analytic solution when $R_0^{ij} > 1$ for any $\{i, j\}$, and in general is better described by a stochastic process, since we will mainly be interested in the time until equilibrium is reached. The result of mutation and migration is that the equilibrium levels will be altered compared to Eq. (3). The major qualitative difference is that strains will persist in compartments where $R_0^{ij} < 1$. When u and m are small, these levels tend to be very low compared to the N_j and K_j^i, and differences in $\{x_j^*, y_{ij}^*\}$ from Eq. (3) are minor. However, as this paper demonstrates, mutation, migration and the relative viral fitness values in different compartments have a major influence on the *time* to reach the equilibrium state.

While in general the migration rates m_{qj} can take on any values, we choose a simple and biologically realistic migration scheme to reduce the number of independent parameters. In this scheme, each pathogen migrates out of its current compartment at rate m. Migrants from a given compartment are then distributed into all four compartments (including the one they came from) proportionally to the compartment sizes, so that larger compartments get more migrants. Therefore, the rate of migration from compartment q to compartment j becomes

$$m_{qj} = m \frac{N_j}{N_{TOT}} \tag{5}$$

where $N_{TOT} = \sum_j N_j$ is the total number of uninfected host cells in the body before infection.

2 Stochastic model

The deterministic viral dynamics model tracking uninfected and infected cells serves well to describe the growth of the infection when the number of cells of any type is large, however, when cell numbers are small, such as when the infection initially starts or when a new strain arises, stochastic effects become important. The deterministic model can be reformulated as a branching process (similar to (3–5)) during these initial stages of infection, since the number of uninfected target cells (x) is approximately constant on this timescale:

$$\begin{aligned} Y_{ij} &\to Y_{ij} + 1 \text{ ... rate: } R_0^{ij} d_y \\ Y_{ij} &\to Y_{ij} - 1 \text{ ... rate: } d_y. \end{aligned} \tag{6}$$

This is a standard birth-death process. Note that there are an infinite number of stochastic processes that reduce to the same deterministic equations, and for some infections, burst-death models (5–7) - where many new infections occur from a single infection nearly simulataneously - may be more appropriate. To keep our model general and to ensure closed form solutions for the probabalistic expressions described below, we have chosen the simplest process.

If a single cell infected with strain i arrives in compartment j where it has $R_0^{ij} > 1$, then the probability it will grow to establish an infection (described by Equation 3) as opposed to going extinct(8) is

$$P_{est}^{ij} = 1 - \frac{1}{R_0^{ij}}. \tag{7}$$

If a single cell infected with strain i arrives in compartment j where it has $R_0^{ij} < 1$, then $P_{est}^{ij} = 0$ but this cell may still infect a few other cells before the infection dies off. The average number of new infections caused by a single infected cell is

$$E[X_{ij}] = \frac{R_0^{ij}}{1 - R_0^{ij}}. \tag{8}$$

Note that Eq. (8) does not count the initial migrant cell, only new infections that occur in compartment j.

This equation makes use of the fact that the probability of producing exactly n offspring is given by

$$\begin{aligned} P(n_{ij}) &= C_n \frac{(R_0^{ij})^n}{(1 + R_0^{ij})^{2n+1}} \\ C_n &= \frac{1}{1+n} \binom{2n}{n}. \end{aligned}$$

where C_n is the n_{th} Catalan number, describing the number of unique infection paths leading to exactly n offspring.

It is important to note that both these equations apply only when there is no previously established infection when the initial cell of strain i arrives. If strain $k \neq i$ is already resident in compartment j, then R_0^{ij} must be replaced by R_0^{ij}/R_0^{kj}, to account for the reduction in target cells (see x^* in Equation 3).

3 Mutation-selection equilibrium

To approximate the probability of resistance via different paths in later sections, we will encounter many expressions that require the frequency at which a rare deleterious mutant exists in a population at equilibrium. Here we describe a method to determine the probability distribution for the number of either one-step or two-step mutants in a compartment where the wild-type (or single mutant) population is at carrying capacity.

We make the following assumptions: The carrying capacity of the resident population is large enough that stochastic fluctuations in size are not important. At each infection event, there is a probability μ that a wild-type infected cell will mutate and instead produce a mutant infected cell. Mutant cells have a infection rate that is reduced by a factor of $1-s$, where $0 < s < 1$ is the cost of the mutation (or the selection coefficient), but die at the same rate d_y. We can assume that $\mu << 1$ so that mutation does not significantly change the equilibrium population size nor the infection rate of the wild-type cells.

3.1 Frequency of single mutants

Here we consider, as an example, mutants resistant to drug 1 that exist before treatment, or in the sanctuary during treatment. The frequency of single mutants can be determined by considering the stochastic process determining the size of the single mutant population (X_1). Let the resident population (in this example, the wild type) be at equilibrium level (K), where the replication rate is equal to the death rate ($d_y K$). We then have the following processes that can stochastically occur in the population:

$$\begin{aligned} X_1 &\rightarrow X_1 + 1 \text{ ... rate: } \mu_1 d_y K \\ X_1 &\rightarrow X_1 + 1 \text{ ... rate: } (1-s_1) d_y X_1 \\ X_1 &\rightarrow X_1 - 1 \text{ ... rate: } d_y X_1 \end{aligned} \tag{9}$$

This is a standard immigration-birth-death process, with immigration rate $I = \mu_1 d_y K$, birth rate $B = (1-s_1)d_y$, and death rate $D = d_y$. The probability generating function for the the size of a population governed by this process (9; 10) is

$$F(z) = \left(\frac{B-D}{Bz-D} \right)^{\left(\frac{I}{B}\right)} \tag{10}$$

and so the PGF for the distribution of the mutant population size is

$$F(z) = \left(\frac{s_1}{1-(1-s_1)z} \right)^{\left(\frac{K\mu_1}{(1-s_1)}\right)} \tag{11}$$

where the probability that there are exactly n mutants can be recovered as $p(n) = \frac{1}{n!}\frac{d^n F}{dz^n}|_{z=0}$. The average number of mutants is

$$E[z] = \frac{dF}{dz}|_{z=1} = K\frac{\mu_1}{s_1}. \tag{12}$$

3.2 Frequency of double mutants

We now assume that one mutation occurs at a rate μ_1 and has cost s_1, while the other has μ_2 and s_2. This situation represents the occurance of double mutants in any compartment before treatment starts or in the sanctuary during treatment. A cell with both mutations can arise by either by a wild-type cell acquiring both mutations simultaneously, or, by a mutant cell with one mutation gaining the other (in either order). The fitness of the double mutant cells is reduced by a factor $(1-s_1)(1-s_2)$.

The frequency of double mutants can be determined by considering the stochastic process determining the size of the single and double mutant populations (X_1, X_2, X_{12}):

$$\begin{aligned}
X_1 &\to X_1 + 1 \text{ ... rate: } \mu_1 d_y K + (1-s_1) d_y X_1 \\
X_1 &\to X_1 - 1 \text{ ... rate: } d_y X_1 \\
X_2 &\to X_2 + 1 \text{ ... rate: } \mu_2 d_y K + (1-s_2) d_y X_2 \\
X_2 &\to X_2 - 1 \text{ ... rate: } d_y X_2 \\
X_{12} &\to X_{12} + 1 \text{ ... rate: } \mu_1\mu_2 d_y K + \mu_1(1-s_2) d_y X_2 + \mu_2(1-s_1) d_y X_1 + (1-s_1)(1-s_2) d_y X_{12} \\
X_{12} &\to X_{12} - 1 \text{ ... rate: } d_y X_{12}
\end{aligned} \tag{13}$$

However, this is no longer a simple immigration-birth-death process and we are not aware of an analytic solution.

An approximate solution can be obtained if we assume that each of the single mutant populations are large enough so that they can also be considered to be at a constant equilibrium level ($K_1 = \mu_1/s_1 K$ and $K_2 = \mu_2/s_2 K$). This approximation is reasonable, because if double mutants are frequent enough to affect treatment failure, then for realistically small values of μ/s , single mutants will be quite frequent.

In this limit, the stochastic process is now:

$$\begin{aligned} &X_{12} \to X_{12} + 1 \text{ ... rate: } \mu_1\mu_2 d_y K + \mu_1(1-s_2)d_y\left(\frac{\mu_2}{s_2}\right)K + \mu_2(1-s_1)d_y\left(\frac{\mu_1}{s_1}\right)K \\ &X_{12} \to X_{12} + 1 \text{ ... rate: } (1-s_1)(1-s_2)d_y X_{12} \\ &X_{12} \to X_{12} - 1 \text{ ... rate: } d_y X_{12} \end{aligned} \tag{14}$$

This is a modified immigration-birth-death process with

$$\begin{aligned} I &= \mu_1\mu_2 d_y K\left(\frac{1}{s_1} + \frac{1}{s_2} - 1\right) \\ B &= (1-s_1)(1-s_2)d_y \\ D &= d_y \end{aligned} \tag{15}$$

and the PGF for the distribution of the double mutant population size is

$$F(z) = \left(\frac{1-(1-s_1)(1-s_2)}{1-(1-s_1)(1-s_2)z}\right)^{\frac{\mu_1\mu_2}{(1-s_1)(1-s_2)}K\left(\frac{1}{s_1}+\frac{1}{s_2}-1\right)} \tag{16}$$

where the probability that there are exactly n mutants can be recovered as $p(n) = \frac{1}{n!}\frac{d^n F}{dz^n}|_{z=0}$. The average number of mutants is

$$E[z] = K\frac{\mu_1\mu_2}{s_1 s_2}. \tag{17}$$

This is the same result that one would derive using a fully deterministic model (for example, see Nowak and May(1)).

Part II
Paths to treatment failure

4 Overview of probability of treatment failure

To obtain a simplified analytic description of the probability distribution of the time to treatment failure in our model, we consider a reduced Markov chain description for the evolution of resistance. The Markov chain reduces the possible number of states of the population using the following assumptions: First, we assume that only one type of cells is present in a compartment at a given time. Second, we assume that when a strain that colonizes a compartment, it instantaneously reaches its carrying capacity. This means that the period when the strain is growing exponentially is ignored. We can make this assumption because exponential growth occurs much faster than evolution (separation of timescales), so the chance that resistance mutations appear when the pathogen is growing exponentially is much lower than the chance that they appear when the infected cells are at carrying capacity. Thus, in this description a compartment is either empty or fully occupied

by only one strain. Transitions between states in this description occur at constant rates given by their average value.

In the simplest case, we will consider two competing paths to resistance: *direct evolution* of double-drug resistance from the sanctuary, or *stepwise evolution* of resistance via the single-drug compartment. For simplicity, **for the rest of the Supplement we will assume there is only one single-drug compartment**, with drug 1, though extension to two is straightforward. We will first derive results without considering the possibility of pre-existing mutations, and then extend our calculations to include this source.

4.1 Acquired resistance only

Let r_{01} be the probability per unit time (the rate) at which the single-drug compartment ("1") is colonized from the sanctuary ("0"). Similarly, r_{12} is the rate of the double-drug compartment ("2") being colonized from the single-drug compartment (once the SDC is colonized), and r_{02} the rate of direct colonization of the double-drug compartment from the sanctuary. Expressions for these rates will be given in the subsequent sections (§5.1-5.3). Assuming that at $t = 0$ only the sanctuary is colonized, and it contains only wild-type cells at carrying capacity, then we can write the probability distribution functions for the time at which the double-drug compartment is colonized via each path as

$$\begin{aligned} P_{02}(t) &= r_{02}e^{-r_{02}t} \\ P_{012}(t) &= \frac{r_{01}r_{12}}{r_{01}-r_{12}}(e^{-r_{12}t} - e^{-r_{01}t}) \end{aligned} \tag{18}$$

where P_{02} refers to the direct path from the sanctuary, and P_{012} refers to the path going through the single-drug compartment. Similarly, the cumulative distribution functions $C(t)$, defined as the probability that the target compartment has already been colonized by a particular time, are written as $C(t) = \int_0^t P(u)du$.

The conditional cumulative distribution function $F(t)$ describes the probability that the double-drug compartment is colonized via a particular path, by a particular time, when both paths are possible. To calculate $F(t)$, we must condition the CDF for each path on the probability that the other path has *not* occurred, resulting in

$$\begin{aligned} F_{02}(t) &= \int_0^t P_{02}(u)(1 - C_{012}(u))du \\ F_{012}(t) &= \int_0^t P_{012}(u)(1 - C_{02}(u))du \end{aligned} \tag{19}$$

4.2 Including pre-existing resistance

At the time that drug treatment is started, there may already be single mutants pre-existing in the single-drug compartment, or double mutants pre-existing in the double-drug compartment, which

can speed up the time to resistance. We term the probability that an individual has an established single mutant population in the single-drug compartment at $t = 0$ as p_{ss} and the same probability for a double mutant in the double-drug compartment as p_{dd}. Then the conditional cumulative distribution functions become

$$\begin{aligned} F_{02}(t) &= (1 - p_{ss}) \left(p_{dd} + (1 - p_{dd}) \int_0^t P_{02}(u)(1 - C_{012}(u))du \right) \\ F_{012}(t) &= p_{ss} + (1 - p_{ss})(1 - p_{dd}) \int_0^t P_{012}(u)(1 - C_{02}(u))du \end{aligned} \tag{20}$$

5 Rates of treatment failure

5.1 Colonization of the single-drug compartment by single resistant mutants

For each single-drug compartment, there are two separate paths by which single drug resistance can arise during treatment, depending on whether mutation or migration from the sanctuary occurs first. Consequently, in the general two-drug case where there are two single-drug compartments, there are four separate paths by which single-drug resistance can happen. Here we present results only considering one single-drug compartment, though the extension to two is simple. Parameter descriptions are given in the main text.

Mutation-migration path In this path a mutant strain is generated in the sanctuary and migrates to the single-drug compartment. The rate at which this path happens is proportional to the carrying capacity of the wild-type population in the sanctuary (number of infected cells at equilibrium), the frequency of the mutant in this population (Equation 12), the migration rate (Equation 5), and the establishment probability (Equation 7) of the mutant in the single-drug compartment:

$$\begin{aligned} r_{01}^{\mu m} &= K_{SAN}^{WT} \frac{\mu_1}{s_1} m_{SAN,SDC} P_{est}^{1,SDC} \\ &= K_{SAN}^{WT} \frac{\mu_1}{s_1} m \frac{N_{SDC}}{N_{TOT}} \left(1 - \frac{1}{R_{WT}(1 - s_1)} \right) \end{aligned} \tag{21}$$

Migration-mutation path In this path a wild-type migrant from the sanctuary goes to the single-drug compartment and gains a mutation. The rate at which this path happens is proportional to the carrying capacity of the wild-type population in the sanctuary (number of infected cells at equilibrium), the migration rate, the size of the wild-type infection in the single-drug compartment before it goes extinct (Equation 8), the mutation rate, and the establishment probability of the mutant.

$$\begin{aligned} r_{01}^{m\mu} &= K_{SAN}^{WT} m_{SAN,SDC} E[X_{WT,SDC}] \mu_1 P_{est}^{1,SDC} \\ &= K_{SAN}^{WT} m \frac{N_{SDC}}{N_{TOT}} \left(\frac{R_{WT}(1 - \epsilon_1)}{1 - R_{WT}(1 - \epsilon_1)} \right) \mu_1 \left(1 - \frac{1}{R_{WT}(1 - s_1)} \right) \end{aligned} \tag{22}$$

Comparison The total rate of colonization is $r_{01} = r_{01}^{\mu m} + r_{01}^{m\mu}$. For most of the parameter ranges we will consider, the rate of the mutation-migration path is much larger than the rate of the migration-mutation path. This is because we consider the cost of the mutation (s) to be relatively small but the drug efficacy to be quite large ($\epsilon \approx 1$), so that the fitness of the mutant in the sanctuary is much larger than the fitness of the wild type in the single-drug compartment. However, if $R_{WT}(1-\epsilon_1) > 0.5$, it is possible for the migration-mutation path to be more important.

5.2 Colonization of the double-drug compartment via the single-drug compartment

There are two separate paths by which double drug resistance can arise from single mutants established the SDC during treatment, depending on whether mutation or migration from the SDC occurs first.

Mutation-migration path In this path a double mutant strain is generated in the single-drug compartment and migrates to the double-drug compartment. We assume the single resistant population in the single-drug compartment is at steady state when mutation occurs. The rate at which this path happens is proportional to the carrying capacity of the single resistant mutant population in the single-drug compartment (number of infected cells at equilibrium), the frequency of the double mutant in this population, the migration rate, and the establishment probability of the double mutant in the double-drug compartment.

$$\begin{aligned} r_{12}^{\mu m} &= K_{SDC}^{1} \frac{\mu_2}{s_2} m_{SDC,DDC} P_{est}^{12,DDC} \\ &= K_{SDC}^{1} \frac{\mu_2}{s_2} m \frac{N_{DDC}}{N_{TOT}} \left(1 - \frac{1}{R_{WT}(1-s_1)(1-s_2)}\right) \end{aligned} \tag{23}$$

Migration-mutation path In this path a single resistant mutant migrant from the single-drug compartment goes to the double-drug compartment and gains a mutation. We assume the single resistant population in the single-drug compartment is at steady state when migration occurs. The rate at which this path happens is proportional to the carrying capacity of the single resistant mutant population in the single-drug compartment (number of infected cells at equilibrium), the migration rate, the size of the single resistant infection in the double-drug compartment before it goes extinct, the mutation rate, and the establishment probability of the double mutant.

$$\begin{aligned} r_{12}^{m\mu} &= K_{SDC}^{1} m_{SDC,DDC} E[X_{1,DDC}] \mu_2 P_{est}^{12,DDC} \\ &= K_{SDC}^{1} m \frac{N_{DDC}}{N_{TOT}} \left(\frac{R_{WT}(1-s_1)(1-\epsilon_2)}{1 - R_{WT}(1-s_1)(1-\epsilon_2)}\right) \mu_2 \left(1 - \frac{1}{R_{WT}(1-s_1)(1-s_2)}\right) \end{aligned} \tag{24}$$

Comparison The total rate of colonization is $r_{12} = r_{12}^{\mu m} + r_{12}^{m\mu}$. For most of the parameter ranges we will consider, the rate of the mutation-migration path is much larger than the rate of the migration-mutation path. This is because we consider the cost of the mutation (s_2) to be

relatively small but the drug efficacy to be quite large ($\epsilon_2 \approx 1$), so that the fitness of the mutant in the SANctuary is much larger than the fitness of the wild type in the single-drug compartment. However, if $R_{WT}(1-\epsilon_2) > 0.5$, it is possible for the migration-mutation path to be more important.

5.3 Colonization of the double-drug compartment directly from the sanctuary

There are three separate paths by which double drug resistance can arise directly from the sanctuary during treatment, depending on the order in which the two mutations are acquired relative to the migration event from the sanctuary.

Mutation-mutation-migration path In this path a double mutant strain is generated in the sanctuary and migrates to the double-drug compartment. The rate at which this path happens is proportional to the carrying capacity of the wild-type population in the sanctuary (the number of infected cells at equilibrium), the frequency of the double mutant in this population, the migration rate, and the establishment probability of the double mutant in the double-drug compartment.

$$\begin{aligned} r_{02}^{\mu\mu m} &= K_{SAN}^{WT} \frac{\mu_1 \mu_2}{s_1 s_2} m_{SAN,DDC} P_{est}^{12,DDC} \\ &= K_{SAN}^{WT} \frac{\mu_1 \mu_2}{s_1 s_2} m \frac{N_{DDC}}{N_{TOT}} \left(1 - \frac{1}{R_{WT}(1-s_1)(1-s_2)}\right) \end{aligned} \tag{25}$$

Mutation-migration-mutation path In this path a single mutant strain resistant to either drug 1 or drug 2 is generated in the sanctuary and migrates to the double-drug compartment, where it gains a second mutation. The rate at which this path happens is proportional to the carrying capacity of the wild-type population in the sanctuary (the number of infected cells at equilibrium), the frequency of the single mutant in this population, the migration rate, the size of the single mutant infection in the double-drug compartment before going extinct, the mutation rate, and the establishment probability of the double mutant in the double-drug compartment. Mutations can occur in either order.

$$\begin{aligned} r_{02}^{\mu m \mu} &= K_{SAN}^{WT} \frac{\mu_1}{s_1} m_{SAN,DDC} E[X_{1,DDC}] \mu_2 P_{est}^{12,DDC} + K_{SAN}^{WT} \frac{\mu_2}{s_2} m_{SAN,DDC} E[X_{2,DDC}] \mu_1 P_{est}^{12,DDC} \\ &= K_{SAN}^{WT} \mu_1 \mu_2 m \frac{N_{DDC}}{N_{TOT}} \left(1 - \frac{1}{R_{WT}(1-s_1)(1-s_2)}\right) \\ &\quad \times \left[\frac{1}{s_1}\left(\frac{R_{WT}(1-s_1)(1-\epsilon_2)}{1-R_{WT}(1-s_1)(1-\epsilon_2)}\right) + \frac{1}{s_2}\left(\frac{R_{WT}(1-s_2)(1-\epsilon_1)}{1-R_{WT}(1-s_2)(1-\epsilon_1)}\right)\right] \end{aligned} \tag{26}$$

Migration-mutation-mutation path In this path a wild-type migrant from the sanctuary goes to the double-drug compartment, where it gains both mutations. The rate at which this path happens is proportional to the carrying capacity of the wild-type population in the sanctuary (the number of

infected cells at equilibrium), the migration rate, the size of the wild-type infection in the double-drug compartment before going extinct, both mutation rates, and the establishment probability of the double mutant in the double-drug compartment. Mutations can occur in either order.

$$\begin{aligned} r_{02}^{m\mu\mu} &= K_{SAN}^{WT} m_{SAN,DDC} E[X_{WT,DDC}] \mu_1 E[X_{1,DDC}] \mu_2 P_{est}^{12,DDC} \\ &\quad + K_{SAN}^{WT} m_{SAN,DDC} E[X_{WT,DDC}] \mu_2 E[X_{2,DDC}] \mu_1 P_{est}^{12,DDC} \\ &= K_{SAN}^{WT} \mu_1 \mu_2 m \frac{N_{DDC}}{N_{TOT}} \left(\frac{R_{WT}(1-\epsilon_1)(1-\epsilon_2)}{1-R_{WT}(1-\epsilon_1)(1-\epsilon_2)} \right) \left(1 - \frac{1}{R_{WT}(1-s_1)(1-s_2)} \right) \\ &\quad \times \left[\left(\frac{R_{WT}(1-s_1)(1-\epsilon_2)}{1-R_{WT}(1-s_1)(1-\epsilon_2)} \right) + \left(\frac{R_{WT}(1-s_2)(1-\epsilon_1)}{1-R_{WT}(1-s_2)(1-\epsilon_1)} \right) \right] \end{aligned} \tag{27}$$

Comparison The total rate of colonization is $r_{02} = r_{02}^{\mu\mu m} + r_{02}^{\mu m\mu} + r_{02}^{m\mu\mu}$. For most of the parameter ranges we will consider, the rate of the mutation-mutation-migration path is much larger than the rate of the migration-mutation-mutation or mutation-migration-mutation path. This is because we consider the cost of the mutations (s_1, s_2) to be relatively small but the drug efficacy to be quite large $(\epsilon_1, \epsilon_2 \approx 1)$, so that the fitness of the mutant in the sanctuary is much larger than the fitness of the wild type in the single or double-drug compartments.

6 Modified rate equations to account for temporal clustering of mutations

We found that the rate expressions used above in the simplified Markov process did a very good job of qualitatively explaining our simulation results, but consistently over-estimated the rate of treatment failure, especially at low mutation rates, high migration rates, and low costs of resistance. Through extensive simulations, we determined that this was due to an approximation inherent in the rate formulas presented in Sections 5.1, 5.2 and 5.3. Because the mutation-migration (or mutation-mutation-migration) path is dominant for all parameter ranges relevant to our study, we focus on describing the issue and correction for this rate.

Equations (21), (24) and (25) assume that mutants (e.g. single mutants in the sanctuary in Eq. (21)) are present at their expected mutation-selection frequency given by Eqs. (12) and (17) at all times. However, in some parts of parameter space, this deterministic approximation leads to a drastic overestimation of the rate of evolution of drug resistance. This overestimate occurs because in reality, the prevalence of mutants varies in such a way that mutants tend to "clump" together temporally. When the total rate of generating single mutants in a compartment is low ($K\mu << 1$), but mutations are not very costly ($s << 1$), then mutants may not be present in the population in most generations, but when they are present, then subcritical but efficient replication may cause them to exist at frequencies much higher than the mutation-selection balance prediction. If, in addition, the eventual probability of migrating and fixing in one of the other compartments is fairly

high for each mutated individual, then the approximations of §5.1-5.3 will be far off, because the probability that at least one of a group of mutants is successful will be a highly non-linear function of the number of mutants. Using the average (or expected) number of mutants will therefore overestimate the rate of adaptation. See Figure S4 for an example of a parameter region where there is less temporal clumping of mutations and migration rates are higher so the simplified Markov process has a much better agreement with the stochastic simulations.

Intuitively, this can be understood as follows. Suppose the expected number of mutations existing in a particular compartment is 1, and each mutant individiual individual has a 10% probability of migrating and establishing in the next compartment. We can demonstrate that adaptation will occur faster if 1 mutant is consistently generated every generation, as opposed to 100 mutants all occuring in one generation, every hundred generations. In the former case, there is overall a 10% change of successful invasion every generation, leading to an expected waiting time for success of 10 generations. In the latter case, there is an $\sim$ 100% chance of succsesful invasion every hundredth generation, leading to an expected waiting time for success of 100 generations. In this case, for a pathogen population trying to adapt, it would be much better to have 1 mutant every generation (assuming expected number of mutants is 1) that has a 10% probability of success, than to have 100 mutants all occurring in one generation, once in a hundred generations even though success is virtually guaranteed in this generation.

Previous work has demonstrated that this effect is important for tunnelling (11) and also for adaptation from standing genetic variation (Fig. 2 of Hermisson and Pennings(12)). Here we adapt the mathematical approach of Weissman et al. (Appendix C) to recalcuate Equations (21), (24) and (25), taking into account this uneven temporal distribution of mutations. We can avoid needing to exactly specify this distribution by instead using a *first-step analysis*, which considers each event that can happen to a single mutant individual and uses this to implicitly calculate the ultimate probability of reaching and establishing infection in a new compartment.

6.1 Colonization of the single-drug compartment by single resistant mutants

We consider first the rate at which the single-drug compartment (SDC) is colonized by single resistant mutants originating in the sanctuary (SAN). We focus on the mutation-migration path, as it is dominant for the entire parameter range of interest in this paper (drug treatment is highly efficacious and mutations have a low fitness cost).

The previous approach involved separately calculating and then multiplying together i) the expected number of secondary mutants generated from each mutation event from the wild type, ii) the probability that each will migrate, and iii) the establishment; instead, this approach calculates this entire process together. We call the overall probability that any single mutant (strain "1", directly resulting from a mutational event or one of its offspring) will migrate from the SAN to the SDC and establish the "rescue probability", $p_{resc}^{1,SAN,SDC}$. Once we know this probability, then the overall

rate of generating single drug resistance is the product of this number and the rate of mutational events from the wild type.

The first-step analysis uses the fact that the the rescue probability for an individual mutant is equal to the probability that this individual itself establishes or that it produces an offspring that establishes. There are four possible first events that can occur to an individual:

- With rate $m_{SAN,SDC}$, it can migrate to the SDC, resulting in rescue with probability $P_{est}^{1,SDC}$
- With rate d_y, it can die, resulting in zero probability of rescue
- With rate $m_{SAN,o} - m_{SAN,SDC}$, it can migrate to another compartment, resulting in zero probability of rescue
- With rate $(d_y + m_{SAN,o})(1 - s_1)$, it can replicate and produce two identical mutant individuals, and the probability that at least one of them is successful is $1 - (1 - p_{resc}^{1,SAN,SDC})^2$.

Here $m_{SAN,o} = \sum_{j \neq SAN} m_{SAN,j}$ is the migration rate to any compartment outside the sanctuary. We also use the fact that the turnover rate of the wild-type population in the SAN at equilibrium is the sum of the death rate (d_y) and the outward migration rate $m_{SAN,o}$, because at equilibrium, input to compartment must equal output from compartment. The replicate rate of the mutant population is reduced by a factor of $1 - s$ compared to the wild type. Because we are using rates rather than probabilities, we can normalize by the sum of the rates to get the expression

$$p_{resc}^{1,SAN,SDC} = \frac{m_{SAN,SDC} P_{est}^{1,SDC} + dy \cdot 0 + (m_{SAN,o} - m_{SAN,SDC}) \cdot 0}{(dy + m_{SAN,o})(2 - s_1)} + \frac{(dy + m_{SAN,o})(1 - s_1)(1 - (1 - p_{resc}^{1,SAN,SDC})^2)}{(dy + m_{SAN,o})(2 - s_1)} \quad (28)$$

This equation can be solved for $p_{resc}^{1,SAN,SDC}$ to give

$$p_{resc}^{1,SAN,SDC} = \frac{-s_1 + \sqrt{s_1^2 + 4(1 - s_1) m_{SAN,SDC} P_{est}^{1,SDC} / (dy + m_{SAN,o})}}{2(1 - s_1)} \quad (29)$$

To calculate the overall rate of this path, $r_{01}^{\mu m}$ we need the rate of mutational events. This is the product of the number of cells turning over each day (the carrying capacity of the SAN, K_{SAN}^{WT} multiplied by the turnover rate) and the mutation rate, u_1. As a result, the rate of invasion of the SDC from the SAN becomes

$$r_{01} \approx r_{01}^{\mu m} = K_{SAN}^{WT} (dy + m_{SAN,o}) \mu_1 p_{resc}^{1,SAN,SDC} \quad (30)$$

6.2 Colonization of the double-drug compartment via the single-drug compartment

Using the same method as above, we get the probability that a mutational event that produces a double mutant (strain "12") in the SDC leads to successful invation of the DDC

$$p_{resc}^{12,SDC,DDC} = \frac{-s_2 + \sqrt{s_2^2 + 4(1-s_2)m_{SDC,DDC}P_{est}^{12,DDC}/(dy + m_{SDC,o})}}{2(1-s_2)} \tag{31}$$

where in this case $m_{SDC,o} = \sum_{j \neq SDC} m_{SDC,j}$, so that

$$r_{12} \approx r_{12}^{\mu m} = K_{SDC}^{1}(dy + m_{SDC,o})\mu_2 p_{resc}^{12,SDC,DDC} \tag{32}$$

6.3 Colonization of the double-drug compartment directly from the sanctuary

We next determine the rate at which the the double-drug compartment (DDC) is colonized by double resistant mutants originating in the sanctuary (SAN). We focus on the mutation-mutation-migration path, for the same reasons discussed above. Because this process involves three steps, we will need to invoke the first-step analysis twice. First, we will need to determine the overall probability that any single mutant (strain "1", directly resulting from a mutational event or one of its offspring) will gain a second mutation and migrate from the SAN to the DDC. We call this rescue probability $p_{resc}^{1,SAN,DDC}$. However, this rescue probability will depend on the probability that any double mutant (strain "12", directly resulting from a mutational event or one of its offspring) will migrate from the SAN to the DDC, $p_{resc}^{12,SAN,DDC}$.

We consider first the probability of rescue starting from a single mutant resistant to drug 1, $p_{resc}^{1,SAN,DDC}$. For a single mutant individual in the SAN, there are four possible first events that can occur:

- With rate $m_{SAN,o}$, it can migrate away, resulting in zero probability of rescue
- With rate d_y, it can die, resulting in zero probability of rescue
- With rate $(d_y + m_{SAN,o})(1-s_1)u_2$, it can replicate and produce one double mutant offspring, and the probability that either the single or double mutant is successful is $p_{resc}^{1,SAN,DDC} + p_{resc}^{12,SAN,DDC}(1 - p_{resc}^{1,SAN,DDC})$.
- With rate $(d_y + m_{SAN,o})(1-s_1)(1-u_2)$, it can replicate without mutating, and the probability that at least one of the resulting single mutants is successful is $1 - (1 - p_{resc}^{1,SAN,DDC})^2$.

We can write the rescue probability as the sum of the probabilities of each first-step event multiplied by probability of rescue conditional upon this first step to get

$$p_{resc}^{1,SAN,DDC} = \frac{m_{SAN,o} \cdot 0 + dy \cdot 0 + (dy + m_{SAN,o})(1 - s_1)u_2(p_{resc}^{1,SAN,DDC} + p_{resc}^{12,SAN,DDC}(1 - p_{resc}^{1,SAN,DDC}))}{(dy + m_{SAN,o})(2 - s_1)} + \frac{(dy + m_{SAN,o})(1 - s_1)(1 - u_2)(1 - (1 - p_{resc}^{1,SAN,DDC})^2)}{(dy + m_{SAN,o})(2 - s_1)} \quad (33)$$

which, when solved for $p_{resc}^{1,SAN,DDC}$, gives:

$$p_{resc}^{1,SAN,DDC} = \frac{-(s_1 + (1 - s_1)u_2(p_{resc}^{12,SAN,DDC} + 1))}{2(1 - s_1)(1 - u_2)} + \frac{\sqrt{(s_1 + (1 - s_2)u_2(p_{resc}^{12,SAN,DDC} + 1))^2 + 4(1 - s_1)^2 u_2(1 - u_2)p_{resc}^{12,SAN,DDC}}}{2(1 - s_1)(1 - u_2)} \quad (34)$$

Rescue could also occur starting from a single mutant resistant instead to drug 2, with a probability $p_{resc}^{2,SAN,DDC}$, which by symmetry is given by

$$p_{resc}^{2,SAN,DDC} = \frac{-(s_2 + (1 - s_2)u_1(p_{resc}^{12,SAN,DDC} + 1))}{2(1 - s_2)(1 - u_1)} + \frac{\sqrt{(s_2 + (1 - s_1)u_2(p_{resc}^{12,SAN,DDC} + 1))^2 + 4(1 - s_2)^2 u_1(1 - u_1)p_{resc}^{12,SAN,DDC}}}{2(1 - s_2)(1 - u_1)} \quad (35)$$

These formulae require knowing $p_{resc}^{12,SAN,DDC}$, which can be calculated using a separate first-step analysis that considers each possible first even that can occur to a double mutant in the sanctuary:

- With rate $m_{SAN,DDC}$, it can migrate to the DDC, resulting in rescue with probability $P_{est}^{12,DDC}$
- With rate d_y, it can die, resulting in zero probability of rescue
- With rate $m_{SAN,o} - m_{SAN,DDC}$, it can migrate to another compartment, resulting in zero probability of rescue
- With rate $(d_y + m_{SAN,o})(1 - s_1)(1 - s_2)$, it can replicate and produce two identical double mutant individuals, and the probability that at least one of them is successful is $1 - (1 - p_{resc}^{12,SAN,DDC})^2$.

This gives us the implicit formula for $p_{resc}^{12,SAN,DDC}$

$$p_{resc}^{12,SAN,DDC} = \frac{m_{SAN,DDC}P_{est}^{12,DDC} + dy \cdot 0 + (m_o - m_{SAN,DDC}) \cdot 0}{(dy + m_{SAN,o})(1 + (1 - s_1)(1 - s_2))} + \frac{(dy + m_{SAN,o})(1 - s_1)(1 - s_2)(1 - (1 - p_{resc}^{12,SAN,DDC})^2)}{(dy + m_{SAN,o})(1 + (1 - s_1)(1 - s_2))} \quad (36)$$

which can be solved to give

$$p_{resc}^{12,SAN,DDC} = -\frac{(1-(1-s_1)(1-s_2))}{2(1-s_1)(1-s_2)} + \frac{\sqrt{(1-(1-s_1)(1-s_2))^2 + 4(1-s_1)(1-s_2)m_{SAN,DDC}P_{est}^{12,DDC}/(dy+m_{SAN,o})}}{2(1-s_1)(1-s_2)}. \quad (37)$$

The overall rate of this path of invasion of the DDC from the SAN, $r_{02}^{\mu\mu m}$, which includes the fact that mutations may occur simultaneously or in either order, is then

$$r_{02} \approx r_{02}^{\mu\mu m} = K_{SAN}^{WT}(dy + m_{SAN,o})(\mu_1 p_{resc}^{1,SAN,DDC} + \mu_2 p_{resc}^{2,SAN,DDC} + \mu_1\mu_2 p_{resc}^{12,SAN,DDC}). \quad (38)$$

6.4 Limiting forms

In particular limits, these modified equations reduce to the expression given in Sections 5.1 - 5.3. By comparing the expressions for $r_{01}^{\mu m}$ and $r_{12}^{\mu m}$ in Equations (30) and (32) to those in (21) and (24), we see that these are equivalent in the limit that $(1-s)p_{mig,est}/s^2 \ll 1$. Here $p_{mig,est}$ is the probability that an individual mutant will migrate to the target compartment before dying or migrating to another compartment. For example, for colonization of the SDC, $p_{mig,est} = m_{SAN,SDC}P_{est}^{1,SDC}/(d_y + m_{SAN,o})$. In this limit, the probability that at least one individual in the lineage of the mutant produced from the wildtype is able to establish infection in a new compartment can be well-approximated by the product of the average lineage size ($1/s$) and the migration-establishment probability ($p_{mig,est}$). Note that this limit does not depend on μ. For the direct path, the conditions that lead to equivalence between $r_{02}^{\mu\mu m}$ given by (38) and (25) are more climplicated, and do depend on μ.

7 Comparison of stepwise versus direct path to acquired double-drug resistance

One way to quantify the influence of single drugs compartments (SDC) on the evolution of drug resistance is to determine the compartment size at which the probability of stepwise evolution becomes equal to the probability of direct evolution in the absence of this compartment. This corresponds to the "crossing point" of the two lines in Fig 2. If the probabilities become equal when the SDC are small relative to the double-drug compartments, this indicates that this extra compartment has a disproportionate influence on the risk of resistance.

Because the analytic calculations described in the previous sections match extremely well with the simulation results (Fig 2 and Fig S4), we can numerically predict the SDC size at the cross point by setting the conditional cumulative distribution functions for the probability of treatment failure (Equation 19) by the direct ($F_{02}(t)$) or stepwise ($F_{012}(t)$) path equal, and solving for N_{SDC}/N_{DDC}. However, we would like to have an expression for this value, to understand its dependence on

the parameter values. While no general closed form solution exists, we can get an approximate expression in two different regimes.

For both regimes we use the simpler expressions given in Section 5, which although neglecting temporal clustering of mutants, only slightly overestimates rates of evolution for the parameter ranges we use, and yields much more comprehensible formulae.

Approximation 1 The first approximation is valid if we look at treatment outcomes when a short enough time (t) has passed so that the prevalence of either single drug resistance or treatment failure is low and all steps are rate-limited ($r_{01}t \ll 1$, $r_{12}t \ll 1$, $r_{02}t \ll 1$). This situation occurs for the results presented in Fig 2a. In this limit,

$$\begin{aligned} F_{02}(t) &\approx r_{02}t \\ &\approx K_{SAN}^{WT}\frac{\mu^2}{s^2}\left(m\frac{N_{DDC}}{N_{TOT}}\right)\left(1-\frac{1}{R_{WT}(1-s)^2}\right)t \\ F_{012}(t) &\approx \frac{1}{2}r_{01}r_{12}t^2 \\ &\approx \frac{1}{2}K_{SAN}^{WT}\frac{\mu}{s}\left(m\frac{N_{SDC}}{N_{TOT}}\right)\left(1-\frac{1}{R_{WT}(1-s)}\right)K_{SDC}^{1}\frac{\mu}{s}\left(m\frac{N_{DDC}}{N_{TOT}}\right)\left(1-\frac{1}{R_{WT}(1-s)^2}\right)t^2 \end{aligned} \tag{39}$$

Setting $F_{02}(t) = F_{012}(t)$, and using the definitions of the carrying capacities (Equation 3), we find that the cross-point occurs when

$$\frac{N_{SDC}}{N_{DDC}} \approx \frac{N_{SDC}}{N_{TOT}} = \left(\frac{1}{2}mN_{TOT}\frac{d_x}{d_y}\left(1-\frac{1}{R_{WT}(1-s)}\right)^2 t\right)^{(-1/2)}. \tag{40}$$

We use the fact that the double-drug compartment comprises the vast majority of the body for all situations we study. Therefore, in this limit, the size of the SDC where the stepwise path to resistance becomes more important than the direct path increases with the pathogen virulence (d_y/d_x), but decreases with the migration rate (m), the total number of uninfected cells before treatment (N_{TOT}, the time of observation (t), and (weakly) with the fitness of the single mutant ($R_{WT}(1-s)$)

Approximation 2 A second approximation may hold for longer times, if the system is in a regime where treatment time is long enough so that most individuals who developed single drug resistance progressed to treatment failure ($r_{12}t \gg 1$), but the other (generally slower) steps remain rate limiting ($r_{01}t \ll 1, r_{02}t \ll 1$. This situation occurs for the results presented in Fig 2b. In this limit,

$$\begin{aligned} F_{02}(t) &\approx r_{02}t \\ &\approx K_{SAN}^{WT}\frac{\mu^2}{s^2}\left(m\frac{N_{DDC}}{N_{TOT}}\right)\left(1-\frac{1}{R_{WT}(1-s)^2}\right)t \\ F_{012}(t) &\approx r_{01}t \\ &\approx K_{SAN}^{WT}\frac{\mu}{s}\left(m\frac{N_{SDC}}{N_{TOT}}\right)\left(1-\frac{1}{R_{WT}(1-s)}\right)t \end{aligned} \tag{41}$$

Setting $F_{02}(t) = F_{012}(t)$, we find that the cross-point occurs when

$$\frac{N_{SDC}}{N_{DDC}} = \frac{u}{s}\left(1 - \frac{1}{R_{WT}(1-s)^2}\right)\left(1 - \frac{1}{R_{WT}(1-s)}\right)^{-1} \approx \frac{u}{s}. \tag{42}$$

We use the fact that the cost of resistance is small ($s \ll 1$) and $R_{WT}(1-s)^2 > 1$. This simpler and more intuitive result demonstrates that the more infrequently mutations occur and the more costly they are, the rarer it is to get double mutants, and the more important the stepwise path involving the SDC is.

Note that for many parameter values and treatment times, neither of these approximations may be appropriate.

8 Including pre-existing resistance

For the main results of the paper, we ignore the effects of pre-existing single or double resistance mutations in compartments containing one or both drugs. However, we present simulation results including this standing genetic variation in Figure 3. Here we present calculations for the probability of pre-existing mutations, and with these expressions and formulation of Section 4.2, we can analytically calculate how standing genetic variation changes the rate of acquiring resistance with and without single-drug compartments.

8.1 Probability of pre-existing single drug resistance in the single-drug compartment

There are two mechanisms by which single drug resistance mutations may colonize the single-drug compartment (SDC) very shortly after treatment begins, without being generated by the sanctuary. When drug treatment starts, there is an existing wild-type infection in all compartments. In the SDC, this infection has a size K_{SDC}^{WT}. A mutation can either exist at mutation-selection balance in this initial population, or, if the drug is not 100% efficacious, it can arise during replication that continues as the wild-type population decays in the presence of the drug.

The relative probabilities of these two paths were considered in an early viral dynamics paper (13), and they found that the probability of pre-existing mutation is always greater than the probability of a newly generated mutation (assuming that the drug treatment results can suppress the wild-type population, i.e. $R_{WT}(1-\epsilon_1) < 1$). They only used deterministic results, and did not consider establishment probabilities. Newer work presented by Alexander and Bonhoeffer(9) revisited this questions through a stochastic viral dynamics framework, and finds more nuanced results - the relative important of *de novo* mutations depends on many parameters of the model - including s, ϵ, d_y/d_x and R_{WT}. Lower drug efficacies and higher costs of the mutation tend to make the contributions of *de novo* mutations greater than pre-existing mutations. Here we summarize the derivations of Alexander and Bonhoeffer as they apply to our system.

Time-dependent establishment probability For both pre-existing and *de novo* single drug mutations arising in the single-drug compartment, we will need to know the establishment probability. When $R_{WT}(1-\epsilon_1) < 1$, which we assume throughout the paper, the wild-type infection in the SDC decays approximately exponentially as uninfected cells recover. The equations describing the dynamics for uninfected cells $x(t)$ and infected cells $y(t)$ are (9)

$$\begin{aligned} x(t) &\approx N_{SDC}(1-(1-1/R_{WT})e^{-d_x t}) \\ y(t) &\approx K_{SDC}^{WT} e^{(g_1(0,s)(e^{-d_x t}-1)+g_2(s)d_x t)} \end{aligned} \tag{43}$$

with the functions g_1 and g_2 given by

$$\begin{aligned} g_1(t,s) &= d_y/d_x e^{-d_x t}(R_{WT}-1)(1-s) \\ g_2(s) &= d_y/d_x(R_{WT}(1-s)-1) \end{aligned} \tag{44}$$

Because the number of available target cells depends on time, so does the effective R_0 of the invading mutant ($R_0^{1,SDC}(t) = R_{WT}(1-s)x(t)/N_{SDC}$) and therefore the establishment probability, $P_{est}^{1,SDC} = 1 - 1/R_0^{1,SDC}$ (§2). Initially, the establishment probability is zero (because $x(0) = N_{SDC}/R_{WT}$ and so $R_0^{1,SDC}(0) = 1-s < 1$), and it increases over time. Alexander and Bonhoeffer(9) derive an expression for $P_{est}^{1,SDC}(t)$ for a mutant that appears at time t,

$$P_{est}^{1,SDC}(t) = \left(1 + \frac{d_y}{d_x} e^{g_1(t,s)} g_1(t,s)^{-g_2(s)} \Gamma(g_2(s), 0, g_1(t,s))\right)^{-1} \tag{45}$$

where the generalized incomplete Gamma function is $\Gamma(z, a_1, a_2) = \int_{a_1}^{a_2} x^{z-1}e^{-x}dx$. Note that $P_{est}^{1,SDC}(t) \neq 0$. Here t refers to the time a mutant appears, not the time at which it establishes, and since each strain has an average lifespan of $1/d_y$ days, it may establish towards the end of its life when infected cell levels have recovered enough that $R_0^{1,SDC} > 0$.

Mutation pre-exists in wild-type population The wild-type population that is initially present in the single-drug compartment before drug treatment may harbor a resistance mutation. As shown in §3.1, the probability generating function for the distribution of the initial single mutant population size is

$$F(z) = \left(\frac{s_1}{1-(1-s_1)z}\right)^{\left(\frac{K_{SDC}^{WT}\mu}{(1-s_1)}\right)}$$

where the probability that there are n mutants can be recovered as $p(n) = \frac{1}{n!}\frac{d^n F}{dz^n}|_{z=0}$, and the average number of mutants is $E[z] = \frac{dF}{dz}|_{z=1} = K_{SDC}^{WT}\mu/s$. The establishment probability of each of these mutants is $P_{est}^{1,SDC}(0)$ (Eq. (45)), and so the overall probability that at least one mutant

establishes an infection is

$$\begin{aligned} p &= \sum_{n=0}^{\infty} p(n)(1-(1-P_{est}^{1,SDC}(0))^n) \\ &= 1 - F(1-P_{est}^{1,SDC}(0)) \\ &= 1 - \left(\frac{s}{1-(1-s)(1-P_{est}^{1,SDC}(0))}\right)^{\left(\frac{K_{SDC}^{WT}\mu}{(1-s)}\right)} \end{aligned} \tag{46}$$

Mutation arises *de novo* from the wild type The wild-type population that is initially present in the single-drug compartment before treatment begins can generate a new resistance mutation during the period when the drug is first administered and the wild-type population is declining. The probability that a new mutation is generated during this decay depends on the product of the rate of new mutations generated and their establishment probability, which are both are time-dependent quantities(9). This product is given by

$$r(t) = K_{SDC}^{WT} d_y R_{WT}(1-\epsilon)\mu \left(1-\left(1-\frac{1}{R_{WT}}\right)e^{-d_x t}\right) e^{(g_1(0,s)(e^{-d_x t}-1)+g_2(s)d_x t)} P_{est}^{1,SDC}(t) \tag{47}$$

where g_1 and g_2 are the same as defined in Equation (44). From $r(t)$, the total probability that a *de novo* mutation single mutant arises in the SDC and establishes is

$$p = 1 - e^{-\int_0^\infty r(t)dt} \tag{48}$$

Comparison The total probability of single mutants establishing in the SDC shortly after treatment initiation, due to standing genetic variation (p_{ss}), is the sum of these two probabilities. When the cost of the mutation is relatively small ($s \ll 1$) but the drug efficacy is quite high ($\epsilon \approx 1$), the chance that these mutants arise from mutation-selection balance is much higher than the chance that they arise *de novo* during drug decay. We tested this for the range of parameter values used in the main text figures.

8.2 Probability of pre-existing double drug resistance in the double-drug compartment

There are three ways to develop resistance in the double-drug compartment that do not involve the other compartments at all. When drug treatment starts, there is an existing wild-type infection in the double-drug compartment. A double mutation can either exist at mutation-selection balance in this initial population, or, if the drug is not 100% efficacious, it can arise during replication that continues as the wild-type population, or pre-existing single resistant mutants, decays in the presence of the drug. Due to our findings (above) that the first path is much more important for single mutations in the SDC, we assume the same is true for double mutants in the DDC, and only present this calculation. The agreement of these approximations with the full simulation results validates this approach

Double resistant mutant pre-exists in wild-type population As described in §3.2, we can approximate the distribution of the number of pre-existing double mutants, and hence the probability that at least one establishes an infection. Using Equation 16, the probability that at least one pre-existing double resistant mutant establishes an infection is

$$p = 1 - \left(\frac{1-(1-s_1)(1-s_2)}{1-(1-s_1)(1-s_2)(1-P_{est}^{12,DDC}(0))} \right)^{K_{DDC}^{WT} \frac{\mu_1 \mu_2}{s_1 s_2}} \tag{49}$$

where $P_{est}^{12,DDC}(0)$ follows the same form as Eq. (45) except that s_1 is replaced with $1-(1-s_1)(1-s_2)$.

We therefore approximate this rate p as the total probability of double mutants establishing shortly after treatment initiation in the DDC, p_{dd}.

Part III
Simulations

9 Overview

We developed a fully stochastic simulation where we keep track of the genotype and location of every infected cell in the body. We explicitly simulate all the events that might occur to an infected cell: replication (representing either division of a bacterial cell or infection of a new cell by a virus), mutation (upon replication), migration and death. Events are chosen with a probability that is proportional to their rate of occurrence. Once an event is executed, time is updated using the total rate of all the possible events that could have occurred. This method for exact stochastic simulation of Poisson processes is known as the Gillespie algorithm.

Rates of replication, death and migration The replication rate of cells of type i in compartment j, $r_{ij}^{replication}$, and the death rate r_{ij}^{death}, are calculated using the deterministic basic viral dynamics model described above (§1). To speed up the simulation of large numbers of cells over long time periods, which we must repeat thousands of times, the model can be additionally simplified to a single equation for the dynamics of infected cells. This is accomplished by assuming that uninfected cells numbers change in parallel to infected cell numbers, without any lags, by setting $\dot{x} = 0$. We then get a reduced model

$$\dot{y_{ij}} = \left[\frac{\lambda_j d_y R_0^{ij}}{\lambda_j + \sum_{l=1}^{n} R_0^{lj} d_y y_{lj}} \right] y_{ij} - d_y y_{ij} \tag{50}$$

where the first term gives $r_{ij}^{replication}$ and the second term tells us that $r_{ij}^{death} = d_y$ for all i, j. The rate of production of uninfected cells in our model is $\lambda_j = N_j d_x$. This approximation in general has very little effect on our results, as lags in target cell recovery or decline matter most when

the total viral population size is changing rapidly in a compartment, which occurs on a timescale much shorter than the evolutionary processes we are interested in. We have validated this using a full simulation tracking infected as well as uninfected cells. However, this lag can influence the fixation probability of pre-existing mutations in drug-treated compartments (§8), and so this approximation was not used for results including standing genetic variation.

The rate of migration out of a compartment does not depend on the type of the cell, so the total outward migration rate is $r_{ij}^{migration} = \sum_{k=0}^{3} m_{jk}$ for all i. As in the main text, m_{jk} is the migration rate from compartment j to compartment k.

10 Simulation algorithm

1. **Calculate the rate of every possible event**
   We denote the rate of an event as α_{ijk} where $i = \{0, 1, 2, 3\}$ corresponds to the genotype of the cell to which the event will occur (wild type, single mutant 1, single mutant 2, double mutant), $j = \{0, 1, 2, 3\}$ corresponds to the compartment where the event will occur (SAN, SDC1, SDC2, DDC) and $k = \{0, 1, 2\}$ corresponds to the type of event (replication,death, migration). We can write $\alpha_{ijk} = n_{ij} r_{ijk}$, where n_{ij} is the number of cells of type i in compartment j and r_{ijk} is the rate at which event k occurs for cells of type i in compartment j. If $k = 0$, $r_{ijk} = r_{ij}^{replication}$, if $k = 1$, $r_{ijk} = r_{ij}^{death}$ and if $k = 2$, $r_{ijk} = r_{ij}^{migration}$. The total rate of possible events that can occur is $\alpha_T = \sum_i \sum_j \sum_k \alpha_{ijk}$.

2. **Determine which event will occur next**
   Draw a random number X from $[0, 1]$. If $X\alpha_T < \alpha_{000}$ the next event will be replication of one wild-type strain in the sanctuary, if $\alpha_{000} < X\alpha_T < \alpha_{000} + \alpha_{001}$, the next event will be death of one wild-type strain in the sanctuary and so on for all the 48 possibilities. We will denote the genotype and compartment of the cell where next event will occur as i' and j' respectively.

3. **Execute event**

   (a) Replication. Draw an additional random number from $(0, 1)$ to determine if the cell mutates to any of the other three genotypes or remains of type i'. Only one mutation event per drug can occur. Increase the number of cells of the type chosen after mutation in compartment j' by 1.

   (b) Death. Decrease the number of cells of type i' in compartment j' by 1.

   (c) Migration. Draw an additional random number from $(0, 1)$ to determine where the cell migrates. Decrease the number of cells of type i' in compartment j' by 1 and increase its number in the target compartment by 1.

4. **Update time** Update the time from t to $t + \tau$ where τ is a number randomly drawn from an exponential distribution with mean $\frac{1}{\alpha_T}$.

These steps are iterated until a maximum time is reached (for Figures 2a, 2b) or until there is colonization of the double-drug compartment (for Figures 2c, 2d, 3 and 4). We assume that the double-drug compartment is colonized and consequently treatments fails if there are more than 10 double-drug resistant mutants in the double-drug compartment. We chose this threshold since the probability that a population of 10 double-drug resistant mutants goes extinct in the double-drug compartment is of the order of 10^{-6} for the parameter values that we use in our simulations.

For results including standing genetic variation we additionaly keep track of the genotype and location of every uninfected cell in the body and explicitly simulate all the events that might occur to both infected and uninfected cells: replication of uninfected cells, infection, mutation upon infection, death of both uninfected and infected cells, and migration of infected cells among different compartments. If treatment does not fail and the number of uninfected cells in the double-drug compartment is restored to the value at carrying capacity by the action of the drug then we switch to simulating only the dynamics of infected cells.

10.1 Initial conditions

10.1.1 Not including standing genetic variation

We assume that when treatment starts the wild-type population is at its carrying capacity in the sanctuary and the other compartments have no infected cells. Thus, the initial number of wild-type strains in the sanctuary is K_{SAN}^{WT}. We also assume that the population is at mutation-selection equilibrium in the sanctuary so we sample the initial number of mutants resistant to drug 1, the initial number of mutants resistant to drug 2 and the initial number of double-drug resistant mutants from Poisson distributions with means $K_{SAN}^{WT}\frac{\mu_1}{s_1}$, $K_{SAN}^{WT}\frac{\mu_2}{s_2}$ and $K_{SAN}^{WT}\frac{\mu_1\mu_2}{s_1 s_2}$ respectively.

10.1.2 Including standing genetic variation

We account for pre-existing mutations by simulating the infection before initiating treatment. We assume that the wild-type starts at its carrying capacity in all the four compartments and simulate the infection for 100 days (since there are no drugs, the R_0 of the wild-type is R_{WT} in all the compartments). We verified that this time is long enough for the population of infected cells to reach mutation-selection balance in all the compartments before treatments starts.

11 Distinguishing paths to resistance evolution in simulations

We distinguish between the direct and the stepwise path to resistance evolution by determining whether the single-drug compartment is already colonized by single-drug resistant mutants when treatment fails. We assume that the single-drug compartment is colonized if it has more than 10 single-drug resistant mutants. We chose this threshold since the probability that a population of 10 single-drug resistant mutants goes extinct in the single-drug compartment is of the order of 10^{-6} for the parameter values that we use in our simulations. When there is more than one single-drug compartment, stepwise evolution of resistance can happen via three different paths: Either of the

single-drug compartments can be colonized before the double-drug compartment or both single-drug compartments can be colonized before the double-drug compartment. We study the relative frequency of the stepwise paths to resistance evolution in Figure 4.

12 Determining the average viral load

The mean viral load is the average of the total number of infected cells over all the time steps in the simulation weighted by the length of each time step. This is $(\sum_i V_i \tau_i)/t_{total}$, where V_i is the total number of infected cells in time step i, τ_i is the length of time step i and $t_{total} = \sum_i \tau_i$ is the total simulation time.

13 Information on figures in the main text

The following parameter values are the same in all figures: $R_{WT} = 4, \epsilon_1 = 0.99, \epsilon_2 = 0.99, d_y = 1$ d^{-1}, $d_x = 0.1$ d^{-1} and $m = 0.1$ d^{-1}.

Figure 2

Figures 2a and 2b

The infection is simulated until the total time has been reached regardless of whether treatment failed or not. The size of the compartment with drug 1, N_{SDC1}, increases along the x-axis and each point is the fraction of the total number of simulated patients that failed via the indicated path (either direct or stepwise). For each value of N_{SDC1} treatment has failed in at least 2000 simulated patients.

Parameters: $s_1 = 0.05, s_2 = 0.05, \mu_1 = 10^{-5}, \mu_2 = 10^{-5}, N_{SAN} = 10^5$ cells, $N_{SDC2} = 0$ cells, $N_{DDC} = 10^7$ cells, Total time: 365 days (Figure 2a), Total time: 3650 days (Figure 2b).

Figures 2c and 2d

We show an example run of a simulated patient where the double-drug compartment is colonized in the absence (Figure 2c) and the presence (Figure 2d) of a single-drug compartment containing drug 1. The mean time to treatment failure over 2000 simulated patients for the parameters in Figure 2c is 1.576×10^5 days and for the parmeters in Figure 2d is 2270 days.

Parameters: $s_1 = 0.05, s_2 = 0.05, \mu_1 = 10^{-5}, \mu_2 = 10^{-5}, N_{SAN} = 10^5$ cells, $N_{SDC2} = 0$ cells, $N_{DDC} = 10^7$ cells, $N_{SDC1} = 0$ cells (Figure 2c), $N_{SDC1} = 5 \times 10^4$ cells (Figure 2d).

Figure 3

We simulate the infection until there is colonization of the double-drug compartment. To capture the trade-off between total drug coverage and the presence of a single-drug compartment, the size of the single-drug compartment with drug 1 increases along the x-axis keeping $N_{SAN}+N_{SDC1}$ constant. We plot both the mean viral load until treatment failure and the fold-increase in the adaptation rate relative to the case when there are no single-drug compartments ($N_{SAN} = 10^5$ cells, $N_{SDC1} = 0$ cells). The adaptation rate is calculated as $\frac{1}{T_f}$ where T_f is the average time to treatment failure over at least 30000 simulations. The fold-increase in adaptation rate relative to $N_{SDC1} = 0$ is shown both for simulations including and not including standing genetic variation.

Parameters: $s_1 = 0.05, s_2 = 0.05, \mu_1 = 10^{-5}, \mu_2 = 10^{-5}, N_{SDC1} = 10^5 - N_{SAN}, N_{SDC2} = 0$ cells, $N_{DDC} = 10^7$ cells.

Figure 4

We assume that there is an additional single-drug compartment where only drug 2 is active. We simulate the infection until there is colonization of the double-drug compartment and study the dependency of the relative frequency of the paths for stepwise resistance evolution on the compartment sizes, the mutation rates and the mutation costs. We consider the paths where only one of the single-drug compartments is colonized before treatment fails. Each point corresponds to the total fraction of patients that failed via the path *SAN* $\rightarrow$ *SDC1* $\rightarrow$ *DDC* relative to the total fraction that failed via the path *SAN* $\rightarrow$ *SDC2* $\rightarrow$ *DDC* out of 6000 replicates.

Figure 4a

We study the effect of asymmetrical compartment sizes on the stepwise paths to resistance evolution by increasing N_{SDC1} along the x-axis while keeping N_{SDC2} constant.

Parameters: $s_1 = 0.05, s_2 = 0.05, \mu_1 = 10^{-5}, \mu_2 = 10^{-5}, N_{SAN} = 10^5$ cells, $N_{SDC2} = 10^4$ cells, $N_{DDC} = 10^7$ cells.

Figure 4b

We study the effect of asymmetrical mutation rates on the stepwise paths to resistance evolution by increasing μ_1 along the x-axis while keeping μ_2 constant.

Parameters: $s_1 = 0.05, s_2 = 0.05, \mu_2 = 10^{-5}, N_{SAN} = 10^5$ cells, $N_{SDC1} = 10^4$ cells, $N_{SDC2} = 10^4$ cells, $N_{DDC} = 10^7$ cells.

Figures 4c

We study the effect of asymmetrical costs of resistance mutations on the stepwise paths to resistance evolution by increasing s_1 along the x-axis while keeping s_2 constant.

Parameters: $s_2 = 0.05, \mu_1 = 10^{-5}, \mu_2 = 10^{-5}, N_{SAN} = 10^5$ cells, $N_{SDC1} = 10^4$ cells, $N_{SDC2} = 10^4$ cells, $N_{DDC} = 10^7$ cells.

Part IV
Supplementary Figures

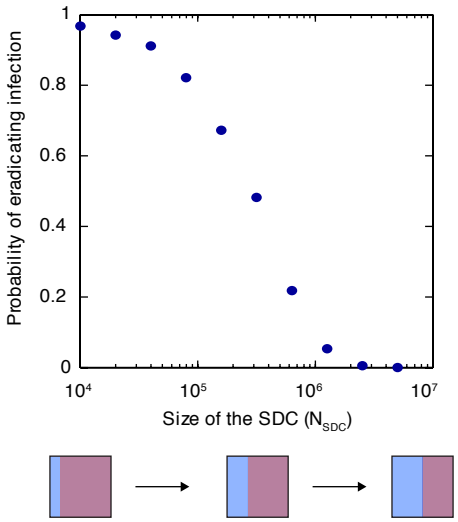

Figure 1: Eradication of an acute disease in the presence of single-drug compartments. The probability that a disease with no sanctuary is eradicated after treatment is plotted as a function of the size of the single-drug compartment with drug 1 (N_{SDC1}), assuming that the sum of the sizes of the SDC1 and the double-drug compartment is constant. Diagrams below the x-axis illustrate the changes in compartment sizes, following the style of Figure 1. The infection is simulated for 100 days before treatment. Treatment starts after this time and is simulated until there are no infected cells in the body (disease eradication) or until the double-drug compartment is colonized. Parameters: $R_{WT} = 4, \epsilon_1 = 0.99, \epsilon_2 = 0.99, d_y = 1\ \text{d}^{-1}, d_x = 0.1\ \text{d}^{-1}, m = 0.1\ \text{d}^{-1}, s_1 = 0.05, s_2 = 0.05, \mu_1 = 10^{-5}, \mu_2 = 10^{-5}, N_{SAN} = 0$ cells, $N_{SDC2} = 0$ cells, $N_{DDC} = 10^7 - N_{SDC1}$. N_{SDC1} changes along the x-axis. Each point corresponds to 3000 simulated patients.

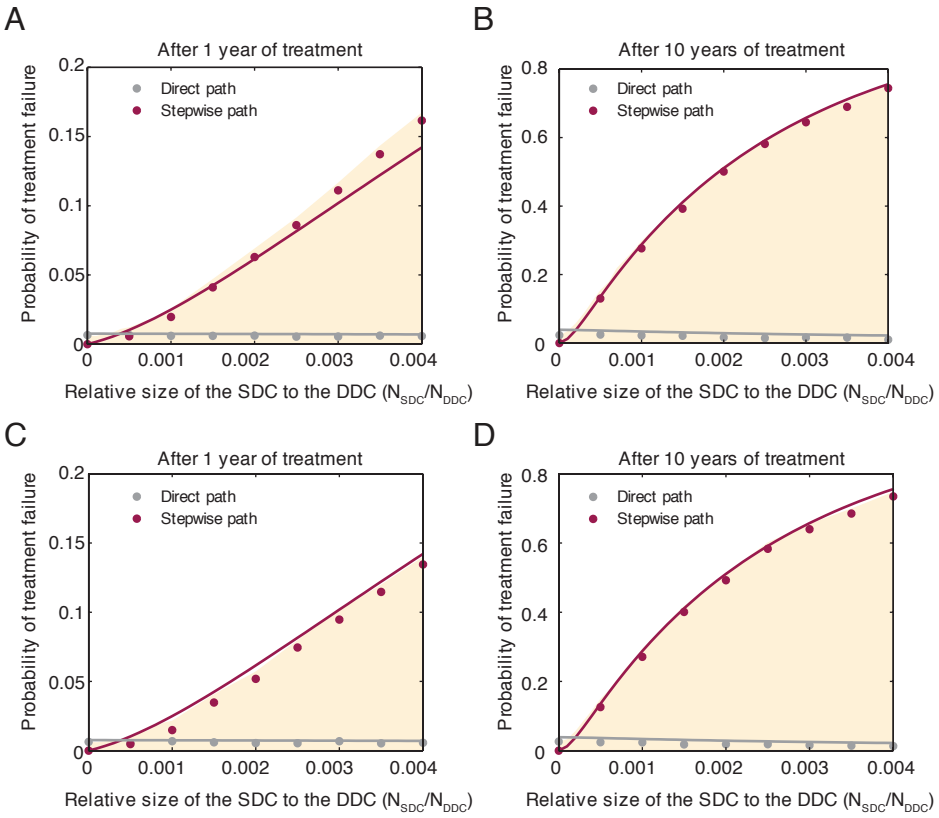

Figure 2: Resistance evolution in the presence of single-drug compartments and pre-existing resistance. (A and B) The shaded area gives the fraction of simulated patients that failed treatment after 1 or 10 years as a function of the size of the single-drug compartment containing drug 1 (SDC1) relative to the size of the double-drug compartment (DDC). The infection in all the compartments is simulated for 100 days before treatment starts. We further indicate whether treatment failure occurred via direct (grey dots) or stepwise evolution (pink dots). Solid lines are analytic calculations (Suppl. Methods §5, 6). (C and D) Same as above, except that backwards migration is not allowed before treatment starts so the number of pre-existing mutants in the single-drug compartment corresponds to the expectation at mutation-selection balance and is not higher because of migration from the double-drug compartment as in A and B. Parameters: $R_{WT} = 4, \epsilon_1 = 0.99, \epsilon_2 = 0.99, d_y = 1\ \mathrm{d}^{-1}, d_x = 0.1\ \mathrm{d}^{-1}, m = 0.1\ \mathrm{d}^{-1}, s_1 = 0.05, s_2 = 0.05, \mu_1 = 10^{-5}, \mu_2 = 10^{-5}, N_{SAN} = 10^5$ cells, $N_{SDC2} = 0$ cells, $N_{DDC} = 10^7$ cells. N_{SDC1} changes along the x-axis for all the figures. For each value of N_{SDC1} treatment has failed in at least 300 simulated patients.

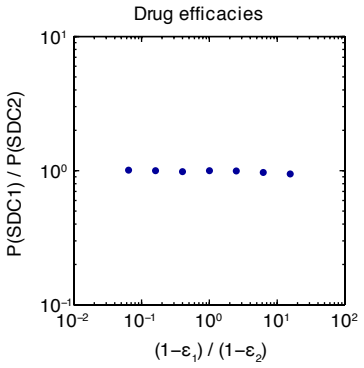

Figure 3: Stepwise resistance evolution in the presence of two single-drug compartments when drug efficacies differ. Fraction of simulated patients that failed via the path where the single-drug compartment with drug 1 is colonized before treatment failure (P(SDC1): *SAN* $\rightarrow$ *SDC1* $\rightarrow$ *DDC*) relative to the fraction that failed via the path where the single-drug compartment with drug 2 is colonized before (P(SDC2): *SAN* $\rightarrow$ *SDC2* $\rightarrow$ *DDC*) as a function of drug efficacies. The x-axis corresponds to the ratio of pathogen fitness in the presence of drug 1 relative to in the presence of drug 2, which is equal to one minus the efficacy of drug 1 (1-ϵ_1) over one minus the efficacy of drug 2 (1-ϵ_2). Parameters: $R_{WT} = 4, d_y = 1$ d$^{-1}, d_x = 0.1$ d$^{-1}, m = 0.1$ d$^{-1}, s_1 = 0.05, s_2 = 0.05, \mu_1 = 10^{-5}, \mu_2 = 10^{-5}, N_{SAN} = 10^5$ cells, $N_{SDC1} = 10^4$ cells, $N_{SDC2} = 10^4$ cells, $N_{DDC} = 10^7$ cells. The total number of simulated patients for each point is 10000.

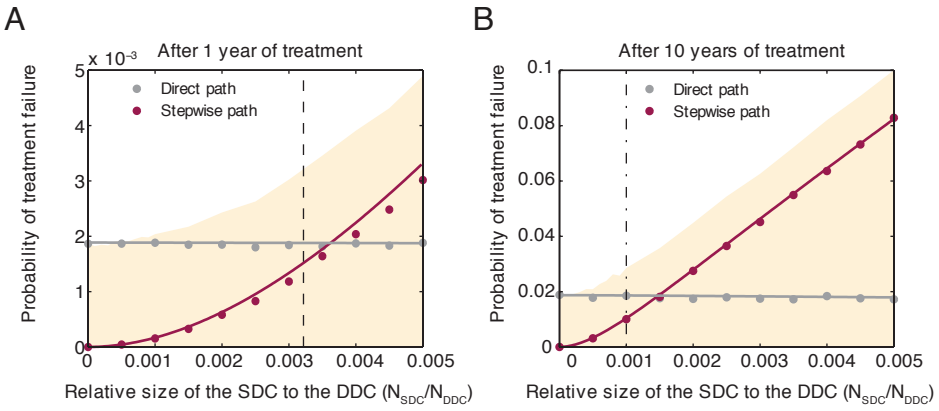

Figure 4: Resistance evolution in the presence of single-drug compartments in a region of the parameter space where the simplified Markov process has a good agreement with the stochastic simulations. The shaded area gives the fraction of simulated patients that failed treatment after 1 or 10 years as a function of the size of the single-drug compartment containing drug 1 (SDC1) relative to the size of the double-drug compartment (DDC). We further indicate whether treatment failure occurred via direct (grey dots) or stepwise evolution (pink dots). Solid lines are simplified analytic calculations (Suppl. Methods §5). The vertical dotted lines are analytical approximations for the point where the stepwise path to resistance becomes more important than the direct path (Supp. Methods §7). Parameters: $R_{WT} = 4, d_y = 1$ d$^{-1}, d_x = 0.1$ d$^{-1}, m = 10^{-3}$ d$^{-1}, s_1 = 0.1, s_2 = 0.1, \mu_1 = 10^{-4}, \mu_2 = 10^{-4}, N_{SAN} = 10^5$ cells, $N_{DDC} = 10^7$ cells. For each value of N_{SDC1} treatment has failed in at least 1000 simulated patients.

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